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SUPPLEMENT TO THE PAPER "ON THE THERMOELASTICITY PROBLEM OF NON-UNIFORM PLATES"†

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Asymptotic expressions for deflections of the order of ε^{-1} ($\varepsilon \to 0$) are constructed in the thermoelastic problem for a non-uniform plate (ε is the characteristic dimension of the periodicity cell). These expressions are then used to obtain the governing equations for the corresponding forces and moments so that the results obtained agreed with the classical results.

A limit problem (a problem of the theory of plates) was obtained in [1] for a thin three-dimensional body with a periodic structure (P_ε is the periodicity cell). The condition that the plate should be thin is formalized in the form $\varepsilon \to 0$. The problem describes the displacements and deflections if a plate of order ε and above (in increasing powers of ε). Classical governing equations were obtained previously in [2] for displacements of the order of ε^0 for the case of when coefficients of thermal expansion of the material of the plate are of the order of ε compared with the other constants. This non-physical assumption (which prevents normal thermal expansion in the plane of the plate from being described) was removed in [1], but non-classical governing relations arose. However, deflections of the order of ε^{-1} were not considered in [1], so that the results obtained in [1, 2] did not completely agree with the classical results [3]. We now consider this in this note.

If $\beta_{ij} \sim \varepsilon^0$ (i.e. the coefficients of thermal expansion are independent of ε), the displacements in the plane of the plate have the same order ε^0. To balance the thermoelastic moments that occur (if the plate if asymmetrical) the normal deflections must be of the order of ε^{-1} (and increase the thinner the plate). We can give appropriate examples of an increase in the deflections although in actual structures these deflections are usually eliminated by an appropriate clamping of the plate. A complete analysis of the problem requires us to consider these terms (as it turns out the classical part of the governing equations corresponds to these).

We will take the expansion for the displacements of the thin three-dimensional body considered in the following form (the notation of [1] is used everywhere henceforth)

$$\mathbf{u} = \varepsilon^{-1} w^{(-1)}(\mathbf{X})\mathbf{e}_3 - y_3 w^{(-1)}_{3,\alpha x}(\mathbf{X})\mathbf{e}_\alpha + u_1(\mathbf{X}) \tag{1}$$

The displacements u_1 are defined by series (2.1) in [1], $\alpha x = \partial/\partial x_\alpha$ ($\alpha = 1, 2$); $\mathbf{X} = (x_1, x_2)$ are the coordinates in the plane of the plate and $y_3 = x_3/\varepsilon$ is the dimensionless coordinate across the plate.

When the expansion is chosen in the form (1) the term of order ε^{-3} in the expansion of the stresses in powers $(-3, -2, \ldots)$ of ε takes the form

$$\sigma_{ij}^{(-3)} = [a_{ijk\alpha}(\mathbf{y})u^{(0)}_{k,\alpha x}(\mathbf{X}) + a_{ijkl}(\mathbf{y})u^{(1)}_{k,ly} - \beta^{(0)}_{ij}(\mathbf{y})\theta(\mathbf{X})] + a_{ij\alpha\beta}(\mathbf{y})y_3 w^{(-1)}_{3,\alpha x\beta x}(\mathbf{X}) \tag{2}$$

The term in square brackets in (2) is identical with the expression for (4.2) in [1]; $jy = \partial/\partial y_j$ ($j = 1, 2, 3$); $\mathbf{y} = \mathbf{x}/\varepsilon$, $\mathbf{x} = (x_1, x_2, x_3)$.

The functions (2) satisfy problem (4.1) of [1]: $\sigma^{(-3)}_{ij,jy} = 0$ in Q_1, $\sigma^{(-3)}_{ij} n_j = 0$ on γ (Q_1 is the region occupied by the plate and γ is its side surface, see [1]). Using the solutions $\mathbf{N}^{\alpha\beta\nu}$ of the cell problems (4.4) [1], we obtain

$$\mathbf{u}^{(1)} = \mathbf{u}^{(1)}_1 + \mathbf{N}^{\alpha\beta 1}(\mathbf{y}) w^{(-1)}_{3,\alpha x\beta x}(\mathbf{X}) \tag{3}$$

where $\mathbf{u}^{(1)}_1$ is given by (4.7) in [1].

Substituting (3) into (2) we obtain

$$\sigma^{(-3)}_{ij} = (a_{ij\alpha\beta}(\mathbf{y})y_3 + a_{ijkl}(\mathbf{y})N^{\alpha\beta 1}_{k,ly}(\mathbf{y}))w^{(-1)}_{3,\alpha x\beta x}(\mathbf{X}) + (a_{ij\alpha\beta}(\mathbf{y}) + a_{ijkl}(\mathbf{y})N^{\alpha\beta 0}_{k,ly}(\mathbf{y}))u^{(0)}_{\alpha,\beta x}(\mathbf{X}) - \\ -(\beta^{(0)}_{ij}(\mathbf{y}) - a_{ijkl}(\mathbf{y})F^{(0)}_{k,ly}(\mathbf{y}))\theta(\mathbf{X}) \tag{4}$$

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Integrating over P_1 ($P_1 = \varepsilon^{-1}P_\varepsilon$ is the periodicity cell in dimensionless variables y) Eq. (4), and the same equation multiplied by y_3, gives the governing relations

$$N_{\gamma\sigma}^{(-3)} = A^0_{\gamma\delta\alpha\beta}u^{(0)}_{\alpha,\beta x} + A^1_{\gamma\delta\alpha\beta}w^{(-1)}_{3,\alpha x\beta x} - B^{(0)0}\theta$$
$$M_{\gamma\sigma}^{(-3)} = A^1_{\gamma\delta\alpha\beta}u^{(0)}_{\alpha,\beta x} + A^2_{\gamma\delta\alpha\beta}w^{(-1)}_{3,\alpha x\beta x} - B^{(0)1}\theta$$
$$(\gamma, \delta, \alpha, \beta = 1, 2) \tag{5}$$

where $A^{\nu+\mu}_{ij\alpha\beta}$, $B^{(\nu)\mu}_{ij}$ are defined by (4.21) and (4.22) in [1]. The equations of equilibrium for the forces $N^{(-3)}_{\gamma\delta}$ and the moments $M^{(-3)}_{\gamma\delta}$ are given by (3.1) and (3.2) [1].

In formula (5) $B^{(0)0}$ are the average coefficients of thermal expansion in the plane of the plate (the analogue of the coefficients of thermal expansion in the plane problem of the theory of elasticity [4–6]), and $B^{(0)1}$ describe the thermoelastic bending moments. As can be seen, the normal deflections $w_3^{(-1)}$ of order ε^{-1} are described by classical-type governing equations, but for the normal deflections $w_3^{(0)}$ of the order of $\varepsilon^{(0)}$ one must use the non-classical equations obtained in [1] (even for the case when $w_3^{(-1)} = 0$).

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